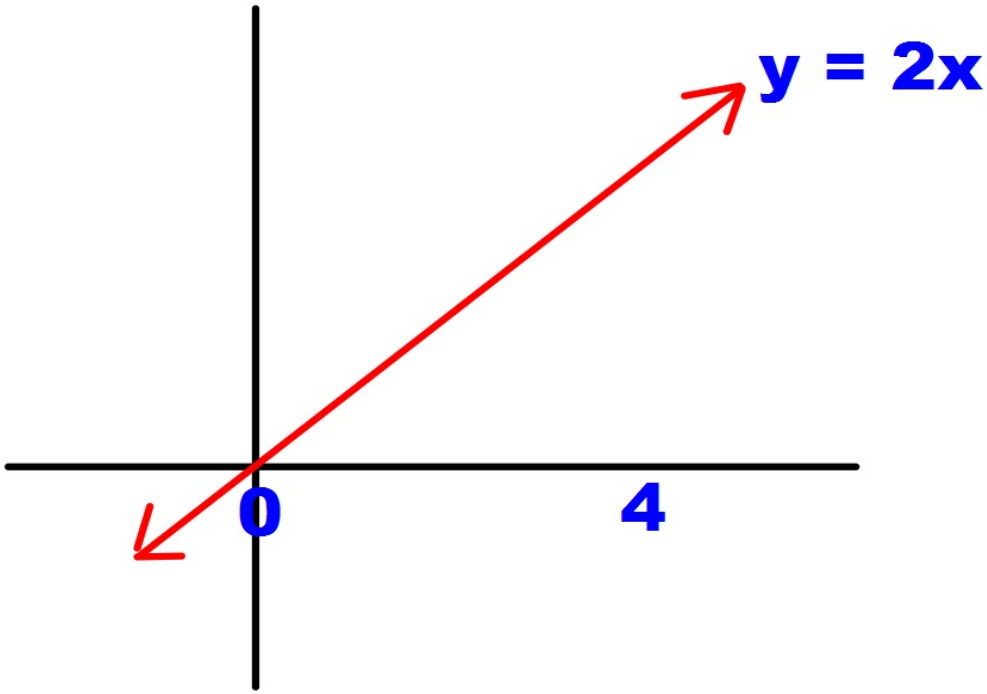
Section 7.3
Volumes

The volume of a solid of known integrable cross section area A(x) from x = a to x = b is the integral of A from a to b.

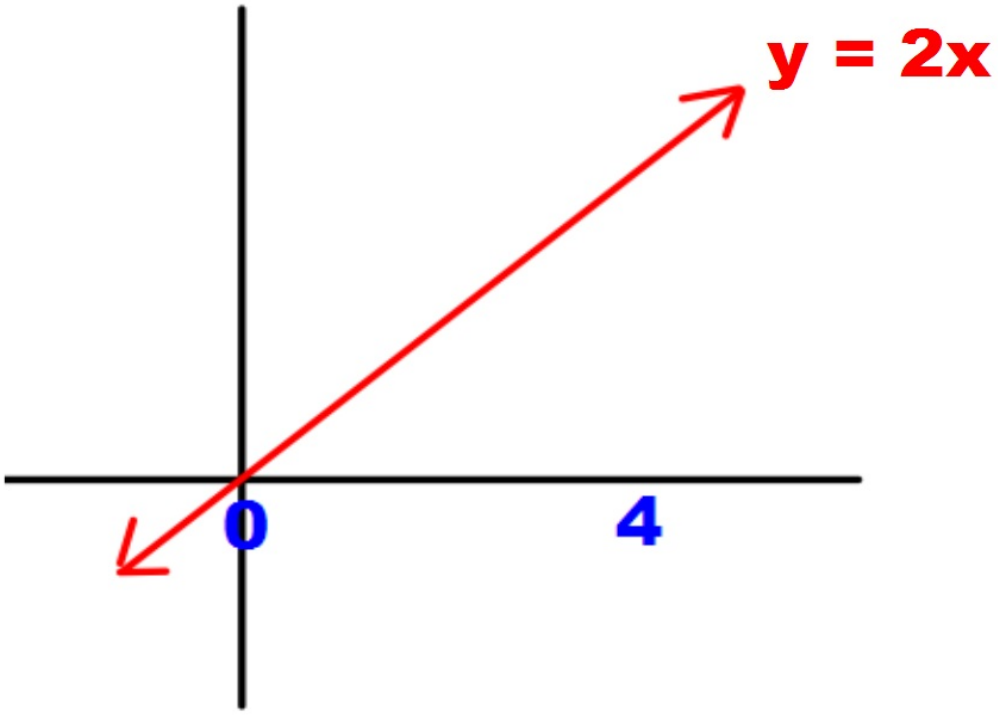
$$V = \int_a^b A(x)\,dx$$

Cross Sections

In the given situation, we can say that the cross sections are squares. Find the volume.



In the given situation, we can say that the cross sections are semicircles. Find the volume.



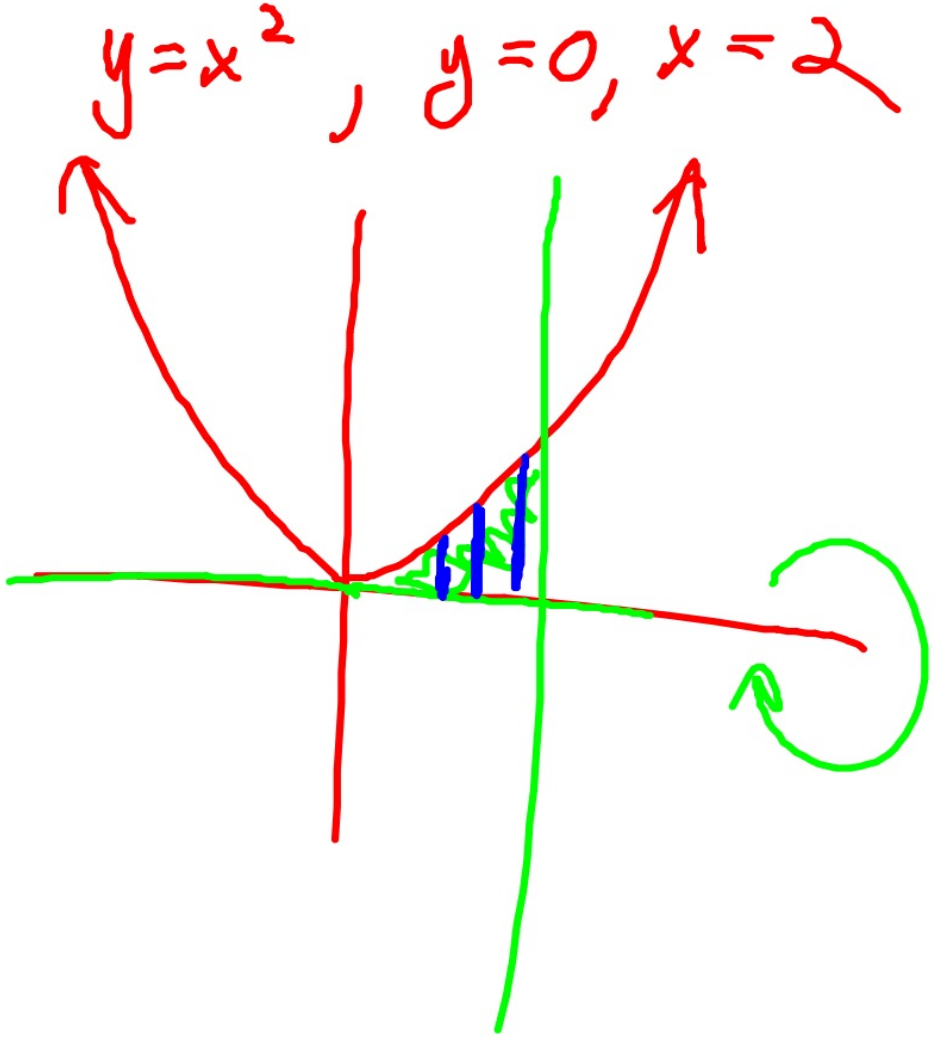
Given the function, $x = y^2$, the cross sections are right triangles where the height is twice the base. Find the volume from 1 to 5.

Given the function, $x = y^2$, the cross sections are right triangles where the height is 2/3 the base. Find the volume from 1 to 5.

Given the two functions, $y = x^2$ and $y = \cos x$.
The cross sections are equilateral triangles.

Given $x^2 + y^2 = 4$. Cross sections are circles.

Given $x^2 + y^2 = r^2$, Cross sections are circles.



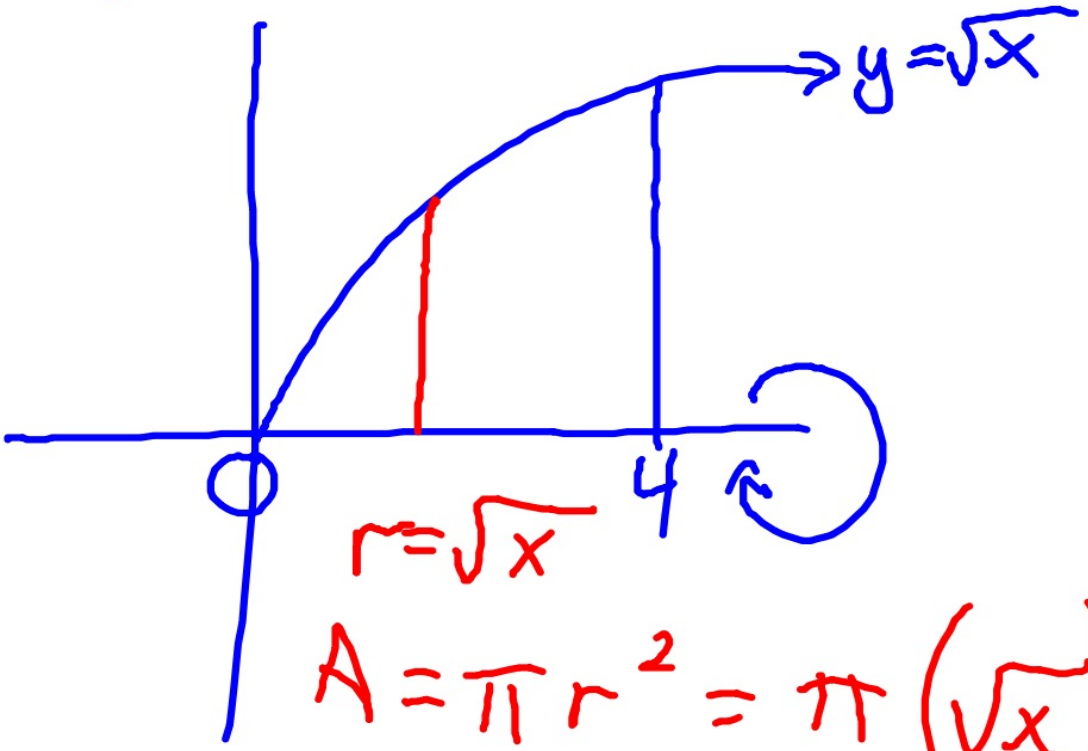
$$r = x^2$$

$$A = \pi r^2 = \pi (x^2)^2$$

$$= \pi x^4$$

$$\int_0^2 \pi x^4 \, dx =$$

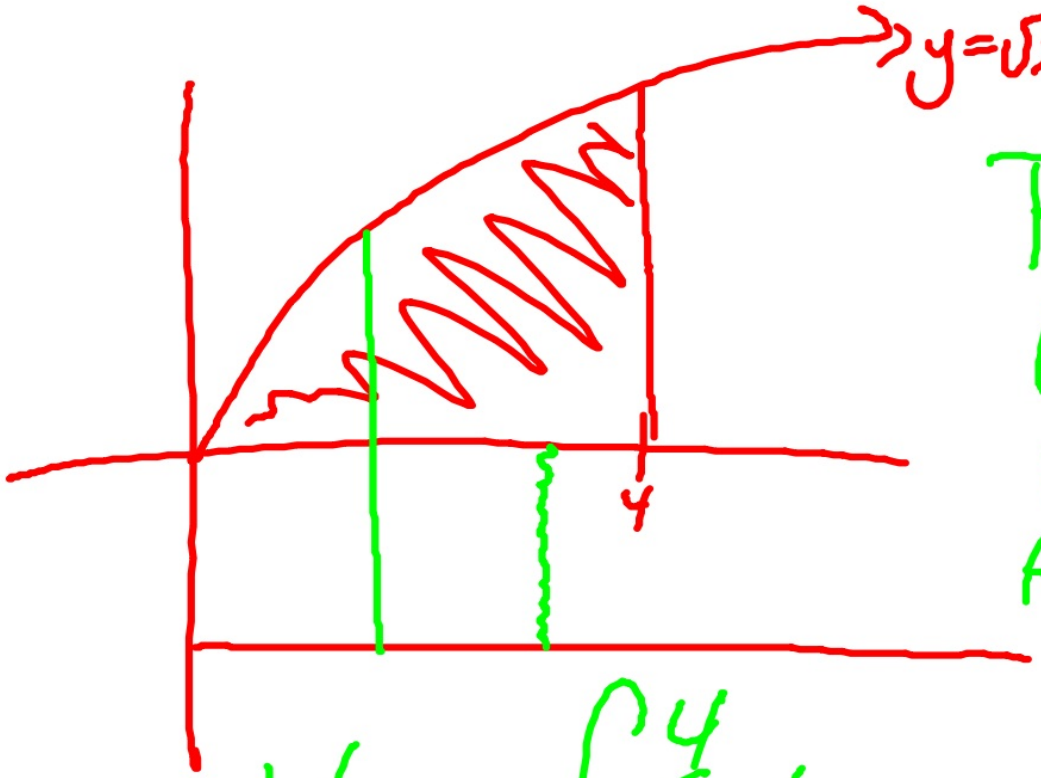
$y = \sqrt{x}$ From 0 to 4.....Known as the Disk Method



$$A = \pi r^2 = \pi \left(\sqrt{x}\right)^2 = x\pi$$

$$\int_0^4 x\pi \, dx =$$

Given $y = \sqrt{x}$ and $y = -2$



Washer Method

$R = \sqrt{x} - (-2) = \sqrt{x} + 2$

$A = \pi(\sqrt{x} + 2)^2$

$r = 0 - (-2) = 2$

$A = \pi 2^2$

$y = -2$

$$V = \int_0^4 \left(\pi(\sqrt{x} + 2)^2 - \pi(2^2)\right) dx$$

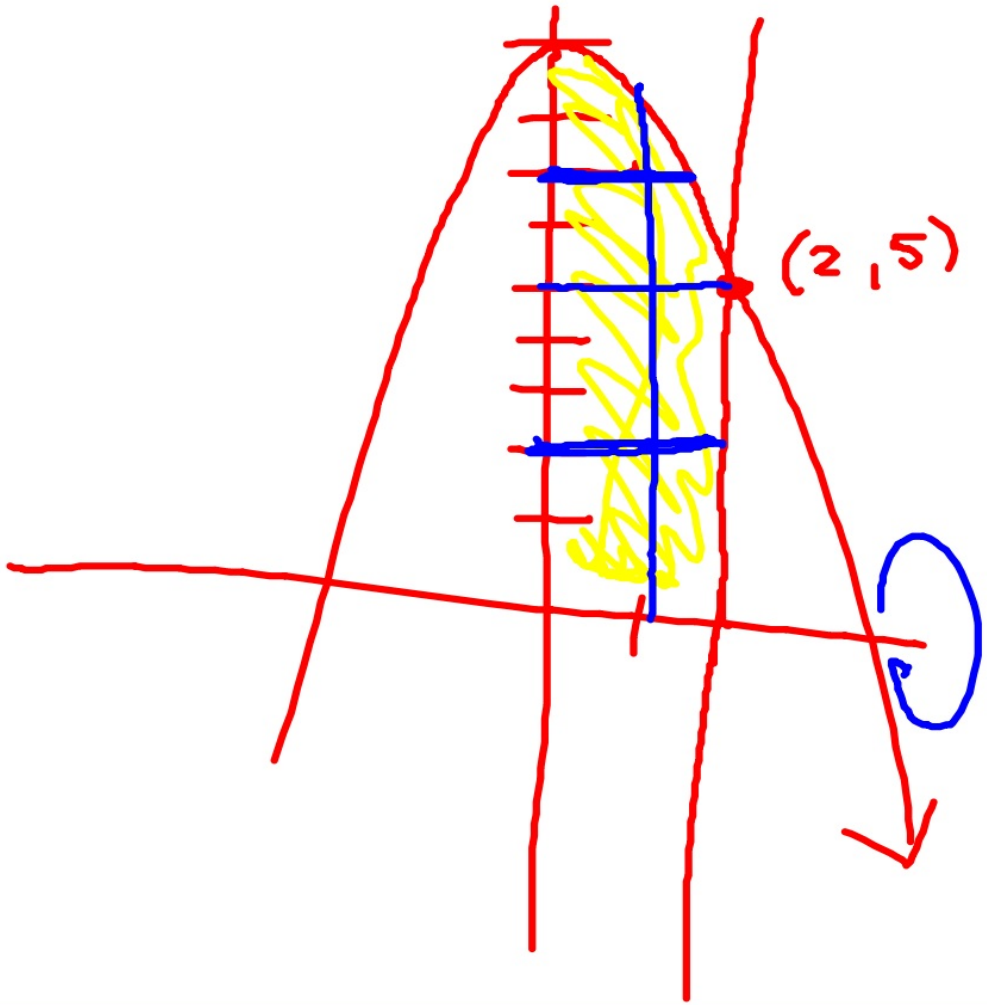
Given y = r, rotated around the x axis, Find the volume of the figure from 0 to h.

$$V = \int_0^h \pi r^2 \, dx = \pi r^2 x \Big]_0^h$$

$$= \pi r^2 h - \pi r^2 (0)$$

$$= \pi r^2 h$$

Revolve the region bounded by $x = 0$, $y = 9 - x^2$ and $x = 2$ and $y = 0$ around the y axis. Then do the same problem around the x axis.



$r = 9 - x^2$

$$\int_0^2 \pi (9 - x^2)^2 \, dx$$

$x = y^3$, $y = (1/4)X$, in first quadrant.

$4y = x$

Revolve around

a.) y - axis

b.) x = 10

c.) x - axis

$R = 4y$

$A = \pi(4y)^2$

$r = y^3$

$A = \pi(y^3)^2$

$$V = \int_0^2 \pi(4y)^2 - \pi(y^3)^2 \, dy$$

Page 406, numbers 2,4,39,43

The region between the graph of $f(x) = 2 + x \cos x$ and the x-axis over the interval $[-2,2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.

Given the $y = \sqrt{x}$ rotated around the $y = 3$.
Find the volume of the figure from 0 to 4.